Lesson 27

USE ANGLE RELATIONSHIPS IN PARALLEL LINES NY-8.G.5

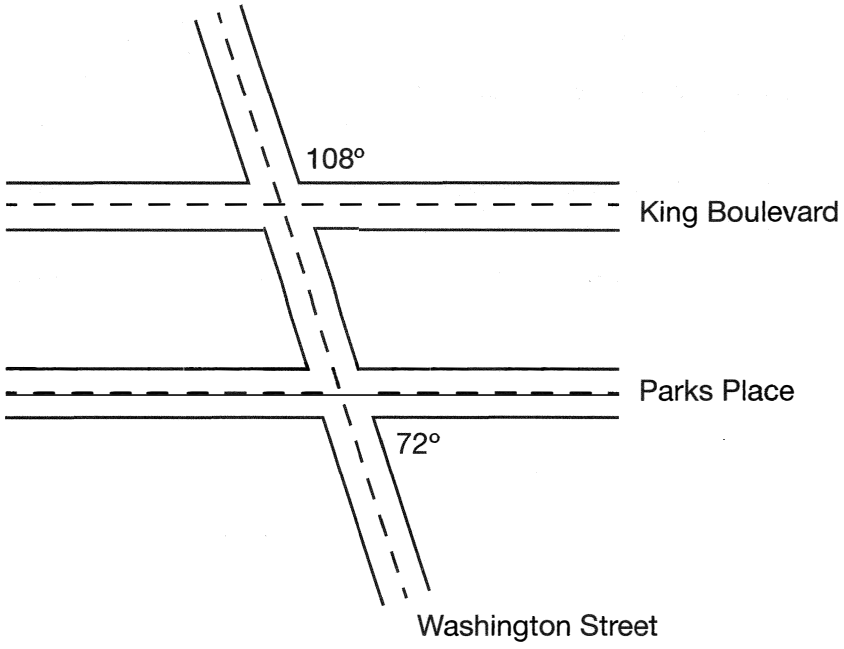
WORDS TO KNOW

parallel lines
perpendicular lines
transversal
congruent
alternate interior angles
alternate exterior angles
corresponding angles

INTRODUCTION

Real-World Connection

Traffic engineers are responsible for the planning, geometric design, and traffic operations of roads, streets, and highways. Jasper is on a team of traffic engineers researching which city roads are best suited for the addition of bike lanes. Jasper wants to know the measure of all of the angles created at each intersection in a diagram of roads in the town. He knows that King Boulevard and Parks Place are parallel. Jasper can use angle relationships in parallel lines to find the angle measures. Let's practice the skills in the **Guided Instruction** and **Independent Practice** and see what the angle measures are at the end of the lesson!



What I Am Going to Learn

- How angles at crossing lines are related
- How to solve problems involving angle measures, where parallel lines are crossed by a non-parallel line

What I May Already Know

- I know how to use supplementary, complementary, adjacent, and vertical angles to solve problems with angle measures.
- I know how to solve algebraic equations.

Vocabulary in Action

Parallel lines are lines in a plane that never intersect.

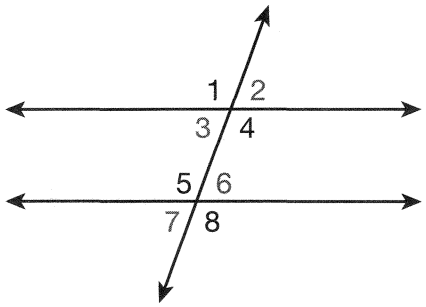
Perpendicular lines are lines that intersect to form right angles.

When a line, called a **transversal**, crosses parallel lines, special pairs of **congruent** angles are formed.

- **Alternate interior angles** are congruent angles between parallel lines and on opposite sides of the transversal: $\angle 3 \cong \angle 6$, $\angle 4 \cong \angle 5$
- **Alternate exterior angles** are congruent angles outside of the parallel lines and on opposite sides of the transversal: $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 7$
- **Corresponding angles** are congruent angles in the same relative position to the transversal: $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, $\angle 4 \cong \angle 8$
- If a transversal is perpendicular to parallel lines, all the angles formed measure 90°.
- You can use angle relationships to find measures of angles formed by a transversal.

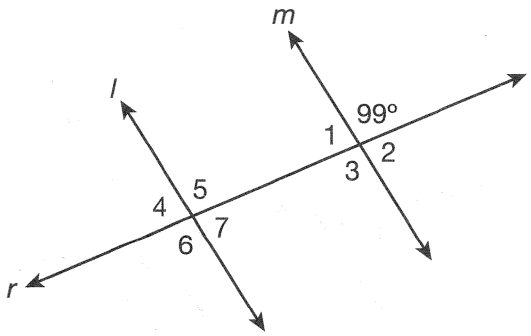
TIPS AND HINTS

The symbol ≅ means "is congruent to."



EXAMPLE

In the figure, line *l* is parallel to line *m*.



Find the measure of ∠5, ∠1, and ∠7.

m∠5

The angle measuring 99° and ∠5 are corresponding angles, so they are congruent.

So, *m*∠5 = 99°.

m∠1

∠1 is supplementary to the angle measuring 99°.

m∠1 + 99° = 180°, so *m*∠1 = 81°

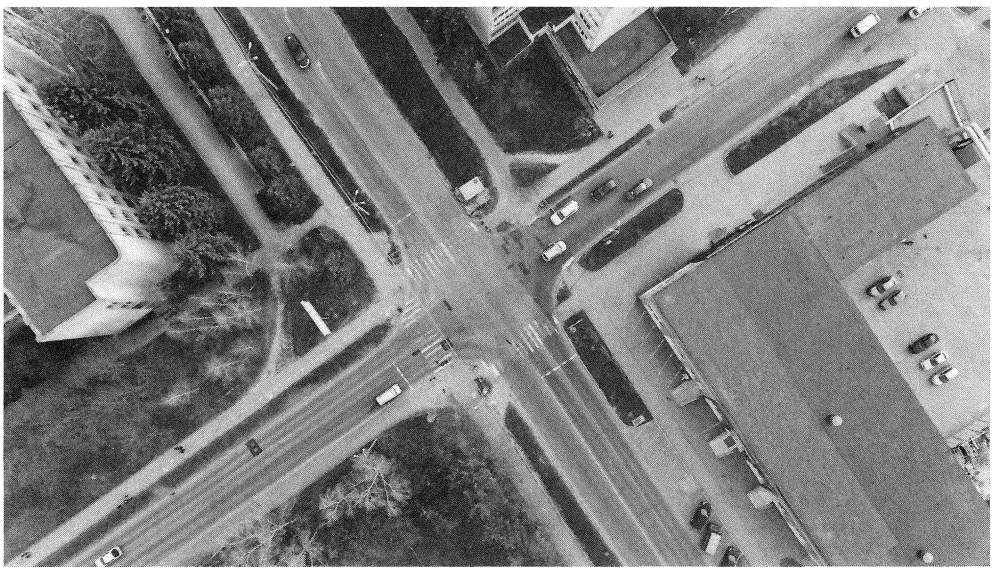
m∠7

∠1 and ∠7 are alternate interior angles, so they are congruent.

m∠7 = 81°

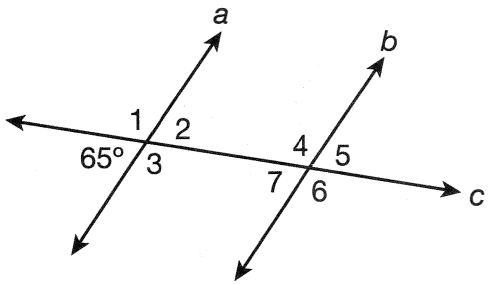
THINK ABOUT IT

Vertical angles are also congruent. This can help with finding angle measures.



GUIDED INSTRUCTION

1. In the figure, line *a* is parallel to line *b*. Find the measures of ∠1, ∠4, and ∠5.



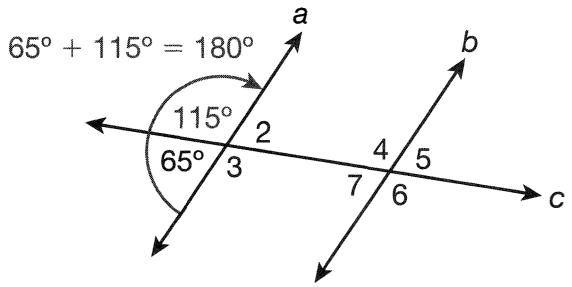
TURN AND TALK

What is the sum of the angle measures of the 4 angles around an intersection?

Step One Find $m\angle 1$.

∠1 and the angle measuring 65° are supplementary, so their sum is 180°.

Subtract to find $m\angle 1$.



180° − 65° = 115°, m∠1 = [ ]°

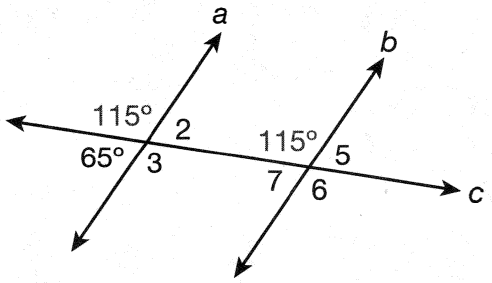
TIPS AND HINTS

Notice that there are only two different angle measures among all the angles, and they are supplementary.

Step Two Find m∠4.

∠4 and ∠1 are corresponding angles, so they are congruent.

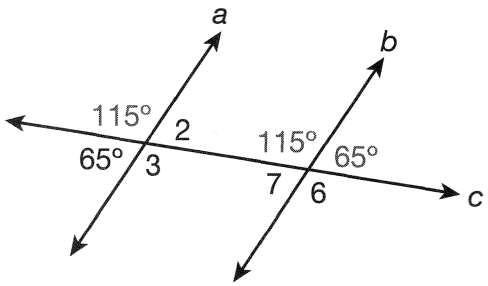
$m\angle 1 = m\angle 4$



m∠4 = 115°

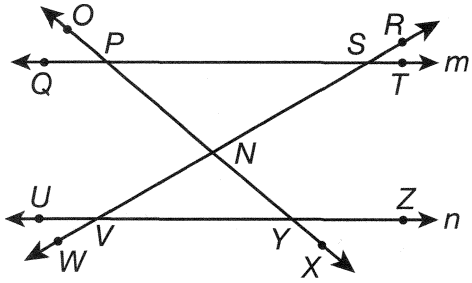
Step Three Find ∠5.

∠5 and the angle measuring 65° are alternate exterior angles, so they are congruent.



m∠5 = [ ]°

2. If $m\angle RST = 4x - 8$ and $m\angle UVW = 3x + 2$, what is x?



Step One Determine the relationship between angles *RST* and *UVW*. There may be more than one way to find the relationship.

Both angles have line *RW* as a ray, so use the transversal *RW* to find angle pairs.

∠*RST* and ∠*NVY* are [ ] angles, so they are congruent.

∠*NVY* and [ ] are vertical angles, so they are [ ].

So, ∠*RST* and ∠*UVW* are [ ].

Therefore, $m\angle RST = m\angle UVW$.

Step Two Write an equation that you can use to solve for x.

$m\angle RST =$ [ ]

$m\angle UVW =$ [ ]

Since $m\angle RST = m\angle UVW$, [ ] = [ ].

Step Three Solve the equation for x.

$4x - 8 = 3x + 2$

$4x - \square - 8 = 3x - \square + 2$

$\square - 8 = 2$

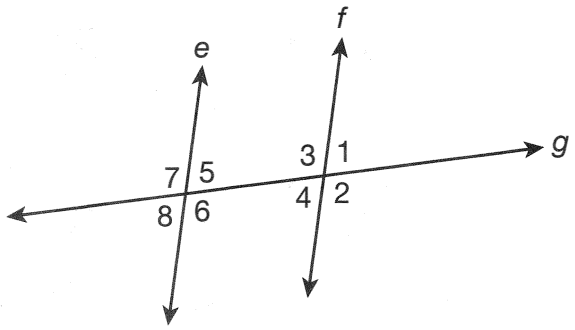
$x - 8 + \square = 2 + \square$

$x = \square$

Step Four If $m\angle RST = 4x - 8$ and $m\angle UVW = 3x + 2$, what is x?

$x = \square$

3. In the figure, line e is parallel to line *f*. Which angle pair is not congruent?



Ⓐ ∠1 and ∠8

Ⓑ ∠8 and ∠4

Ⓒ ∠4 and ∠7

Ⓓ ∠7 and ∠6

Learning Together

Working in a small group, use tape to lay out two 10-foot parallel lines on the floor. Place the parallel lines about 4 feet apart. Then add a transversal that cuts through both parallel lines. Have one group member stand at one angle and call out another member's name and a type of angle, such as congruent, vertical, alternate interior, alternate exterior, or corresponding. The named member should stand at an angle that matches the called-out type of angle. Once in this position, this second member should call out another member name and type of angle. Continue in this fashion until all members are on an angle.

How Am I Doing?

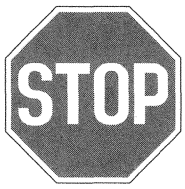
What questions do you have?

How can you find angle measures for the angles formed when transversals cross parallel lines, given one of the angle measures?

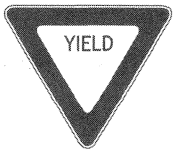
There are many situations where you see parallel lines in everyday life.

What is one situation where these lines are crossed by another line?

Circle the sign that shows how you are doing with the skill.



I am stuck.

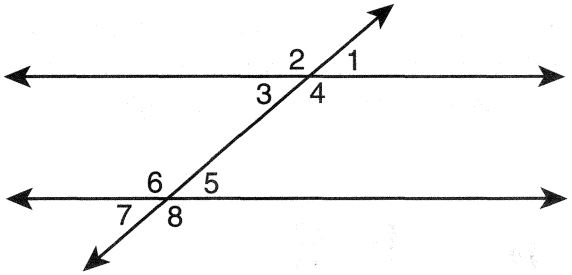


I almost have it.

I understand the skill.

INDEPENDENT PRACTICE 1

1 The image below shows two parallel lines cut by a transversal.



Which angle is supplementary to $\angle 2$?

A $\angle 4$

B $\angle 5$

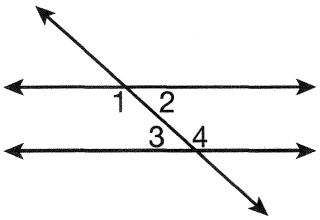
C $\angle 6$

D $\angle 8$

THINK ABOUT IT

Part of the given information for this problem is that the two lines that look parallel are actually parallel. When calculating angle measurements, why is it critical to know for sure that two lines are actually parallel rather than just assuming they are parallel?

2 The image below shows two parallel lines cut by a transversal.



What is the measure of angle 1 if the measure of angle 2 is 42°?

A 38°

B 42°

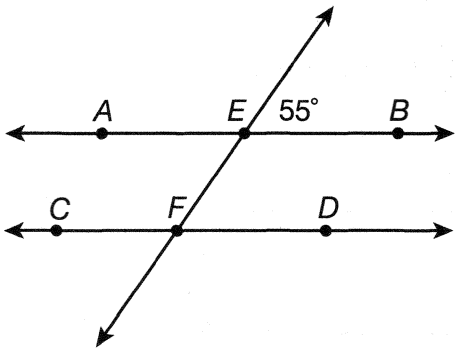
C 138°

D 142°

TIPS AND HINTS

A transversal is simply a line that cuts through two or more other lines.

3 The image below shows two parallel lines cut by a transversal.



What is the measure of angle *CFE*?

A 45°

B 55°

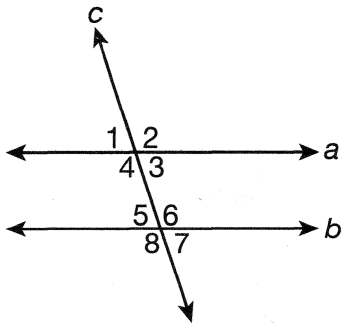
C 125°

D 180°

THINK ABOUT IT

This image provides one angle measurement. With that one measurement, can you find all the other angle measurements?

4 The image below shows two parallel lines cut by a transversal.



Angle 1 has a measure of 72°. What is the measure of each of the other angles?

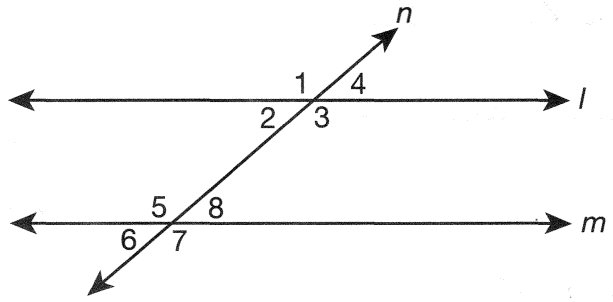
Explain your answer.

SKETCH IT

To see a pattern and make the angle relationships easier to understand, create eight drawings of the image for this problem. On each drawing, circle one of the angles. Then, in relation to that angle, label each of the other angles as either a vertical angle (v), alternate interior angle (ai), alternate exterior angle (ae), or corresponding angle (c).

INDEPENDENT PRACTICE 2

1 The image below shows two parallel lines cut by a transversal.



Which angle has the same measure as angle 1?

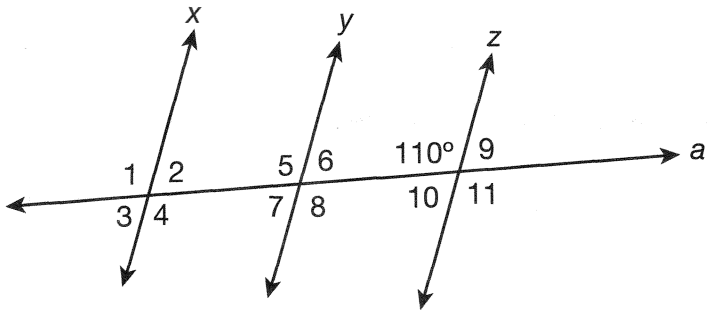
A angle 2

B angle 4

C angle 5

D angle 6

2 The image below shows three parallel lines cut by a transversal.



Which term describes the relationship between ∠1 and ∠6?

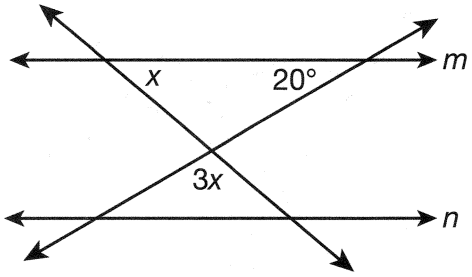
A alternate exterior angles

B alternate interior angles

C corresponding angles

D supplementary angles

3 The image below shows two parallel lines cut by two transversals.



Given that line *m* is parallel to line *n*, what is the value of *x*?

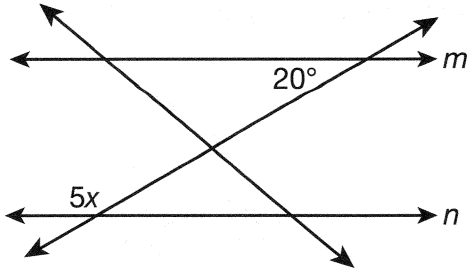
A 20°

B 30°

C 40°

D 50°

4 The image below shows two parallel lines cut by two transversals.



What is the value of *x*?

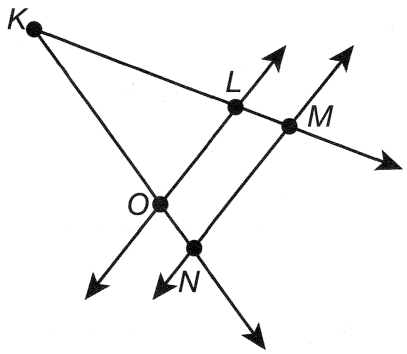
A 20°

B 32°

C 36°

D 160°

5 The figure below shows parallel lines cut by two transversals.



If $m\angle KLO = 3x + 12$ and $m\angle NMK = 4x - 9$, which equation can be used to find x?

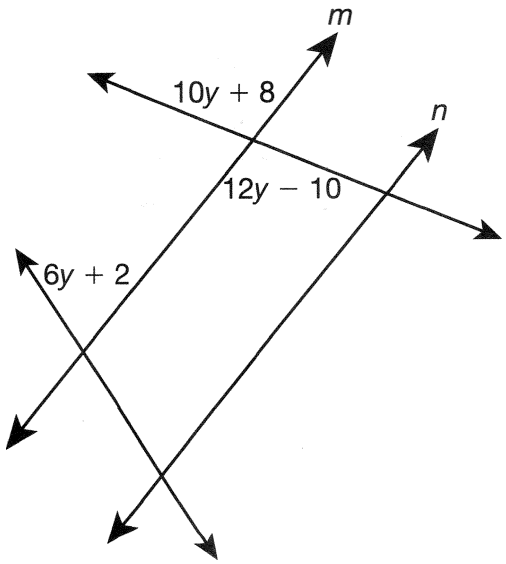
A $3x + 12 = 4x - 9$

B $(3x + 12) + (4x - 9) = 180$

C $(3x + 12) - (4x - 9) = 180$

D $(3x + 12) + (4x - 9) = 90$

6 The figure below shows parallel lines cut by two transversals.



What is the value of y?

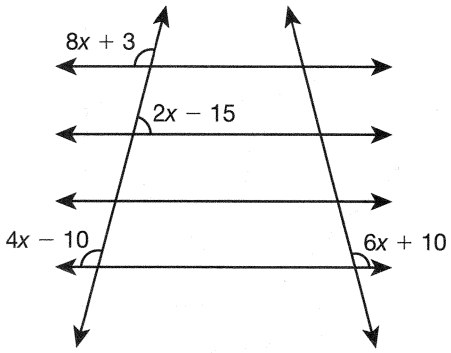
A 1

B 2

C 9

D 12

7 The figure below shows four parallel lines intersected by two lines.



Which equation can be used to find the value of x?

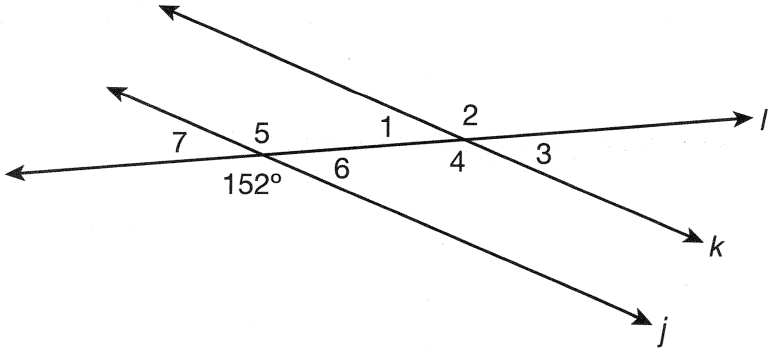
A $2x - 15 = 8x + 3$

B $2x - 15 = 4x - 10$

C $8x + 3 = 4x - 10$

D $8x + 3 = 6x + 10$

8 The image below shows two parallel lines cut by a transversal.



What is the measure of $\angle 2$?

Answer _____

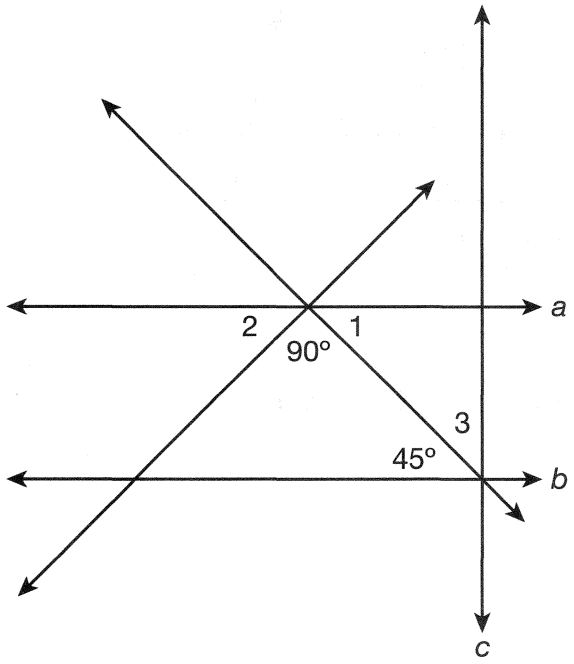
Explain your answer using angle relationships.

_____

_____

_____

9 In the figure below, line *a* is parallel to line *b* and perpendicular to line *c*.



Find the measure of ∠1, the measure of ∠2, and the measure of ∠3.

Answer $m\angle 1 =$ _____, $m\angle 2 =$ _____, $m\angle 3 =$ _____

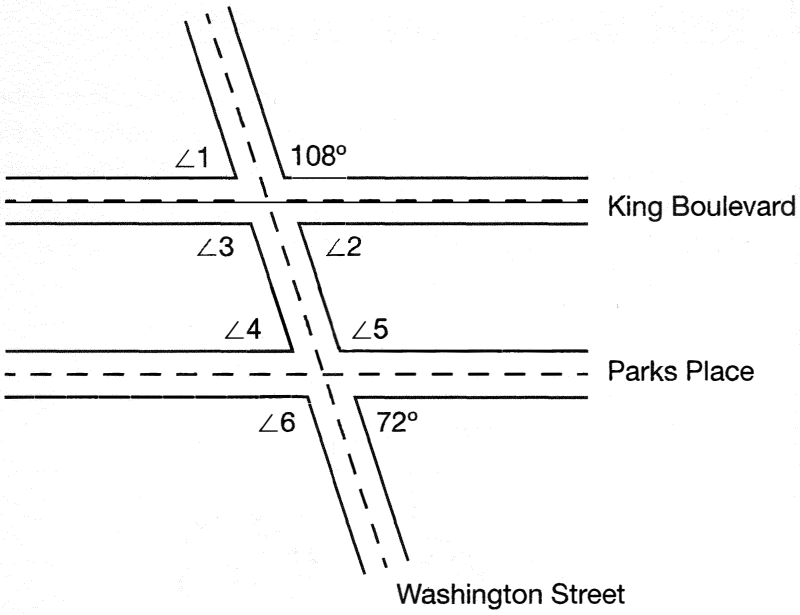
Explain your answers using angle relationships.

EXIT TICKET

NY-8.G.5

Now that you have mastered solving problems with angles and parallel lines and transversals, let's solve the problem in the **Real-World Connection**.

Jasper wants to know the measure of all of the angles created at each intersection in a diagram of roads in the town. He knows that King Boulevard and Parks Place are parallel. Jasper can use angle relationships in parallel lines to find angle measures. King Boulevard and Parks Place are parallel and Washington Street is a transversal.



Use known angle measures and identify congruent and supplementary angles to help Jasper solve for the unknown angle measures.

$m\angle 2 =$ ☐ $m\angle 4 =$ ☐ $m\angle 1 =$ ☐

$m\angle 6 =$ ☐ $m\angle 3 =$ ☐ $m\angle 5 =$ ☐